Lattice Calculations of Nucleon Form Factors

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[arXiv:0907.4194 [hep-lat] (Phys. Rev. D81:034507, 2010)]
[arXiv:1001.3620 [hep-lat]]

1. Lattice simulations with chiral quarks
2. Vector Form Factors
3. Axial Form Factors
4. Outlook

1 Lattice simulations with chiral quarks

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Nucleon Form Factors

- Vector form factors: Dirac F_1 and Pauli F_2

$$\langle N(p+q)|\bar{q}\gamma^\mu q|N(p)\rangle = \bar{u}_{p+q}\Big[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2\Big]u_p$$

electron scattering off protons&nuclei

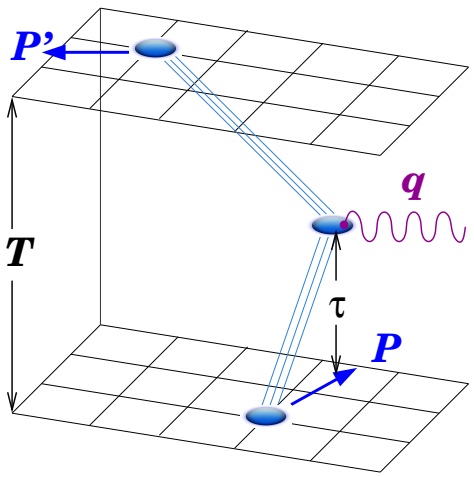
- Axial form factors

$$\langle N(p+q)|\bar{q}\gamma^\mu\gamma_5 q|N(p)\rangle = \bar{u}_{p+q}\Big[\gamma^\mu\gamma_5 G_A + \frac{q^\mu}{2M}\gamma_5 G_P\Big]u_p$$

ν scattering off protons&nuclei, charged pion electroproduction, muon capture

Focus on *systematically clean* isovector $(u-d)$ form factors

Nucleon Structure on a Lattice



Nucleon Structure in Lattice QCD:

- 2+1 flavors, $m_u = m_d = \hat{m} \ll m_s$
- spacelike $0 \leq Q^2 \lesssim 1$ GeV^2
- $a = 0.084 \ldots 0.124$ fm
- $L > m_\pi^{-1}$

Current and past simulations:

- 300 MeV $\lesssim m_\pi \lesssim 600$ MeV with chiral *valence* quarks
- 300 MeV $\lesssim m_\pi \lesssim 400$ MeV with chiral *valence & sea* quarks
- (Preliminary) down to $m_\pi = 200$ MeV with non-chiral (Wilson-Clover) quarks

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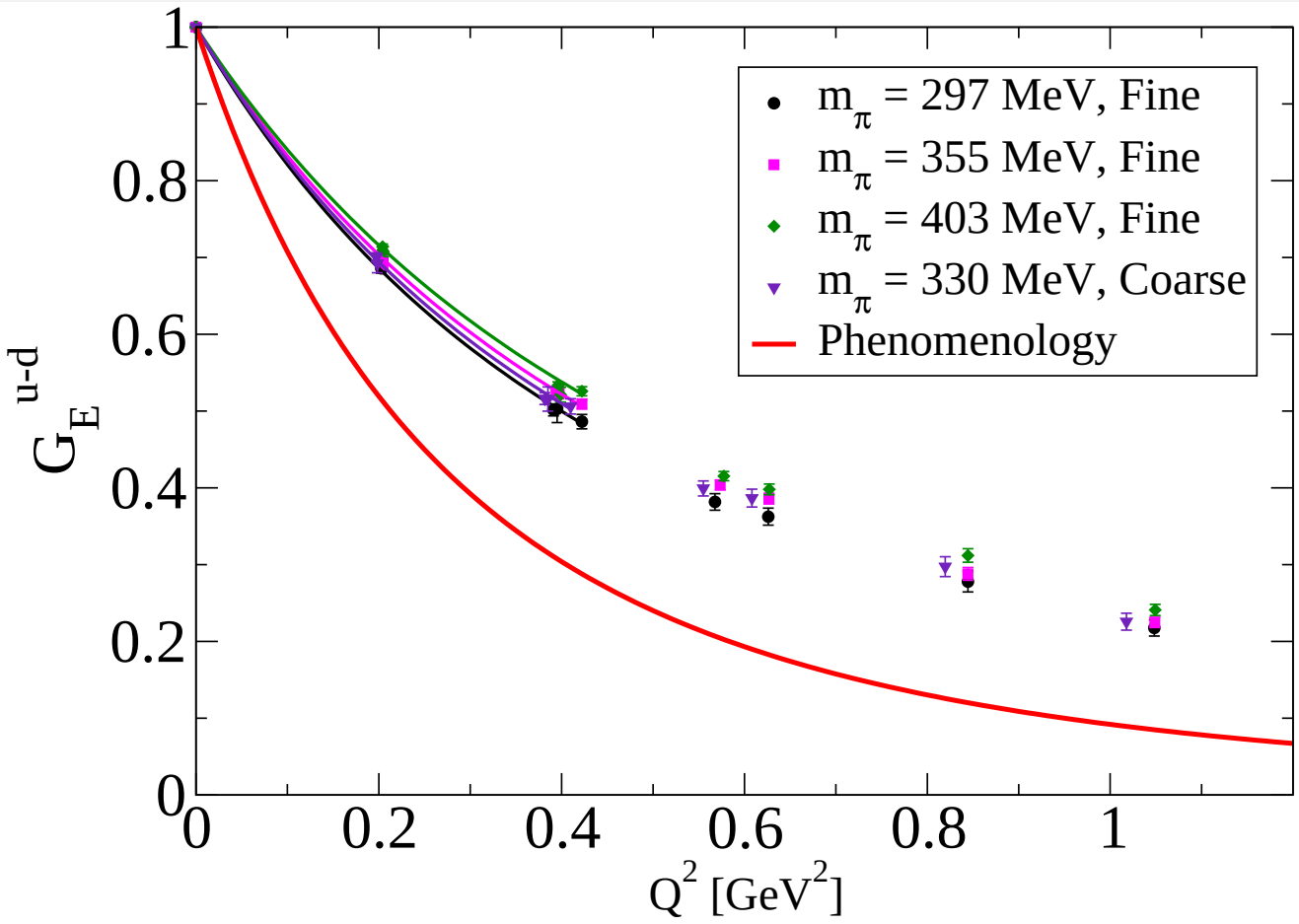
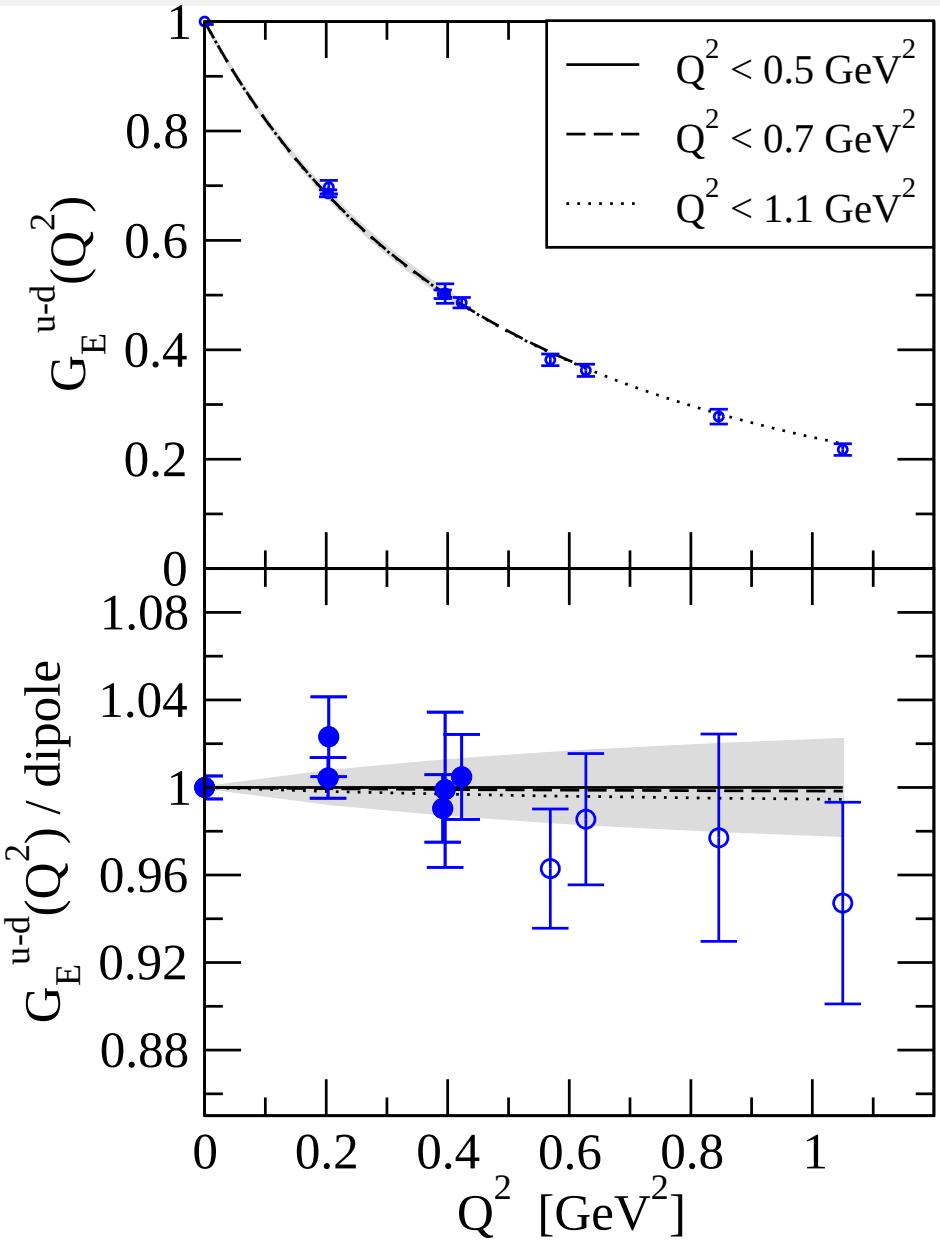
Electric Form Factor G_E^{u-d} vs. Q^2



$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2) \sim \langle N(+\tfrac{\vec{q}}{2}) | Q | N(-\tfrac{\vec{q}}{2}) \rangle$$

Fit to experimental data: [J. J. Kelly '04]

Dipole Fits to G_E^{u-d} ($m_\pi = 297$ MeV)

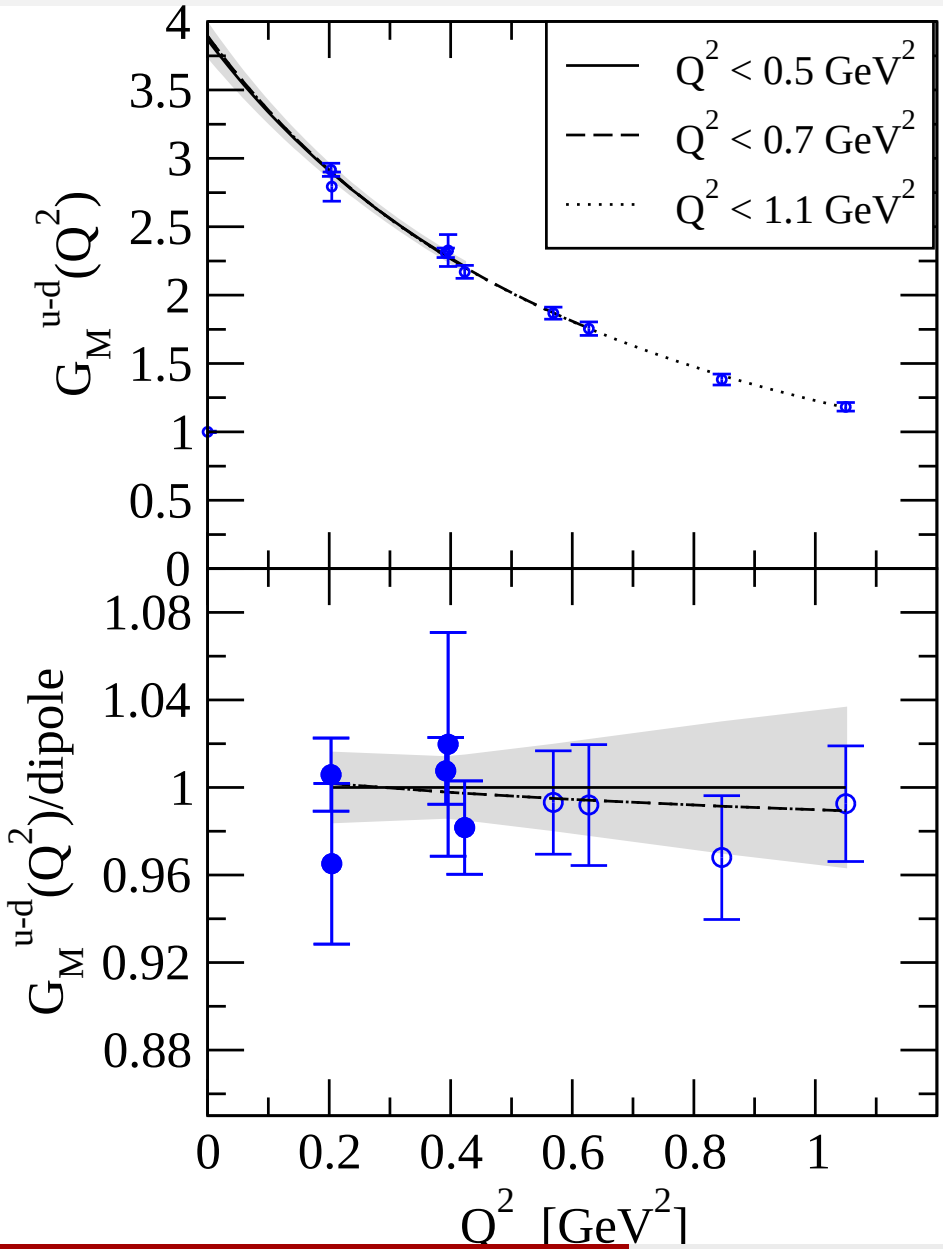


- Dipole fits $G_E(Q^2) = \frac{1}{(1+Q^2/M_D^2)^2}$
- Cut-off $Q^2 \leq 0.5$ GeV^2
- Results are consistent for higher cutoffs

$M_D^2 = 0.97 \ldots 1.07$ GeV^2

$(M_D^2)_{\text{exp}} = 0.71$ GeV^2

Dipole Fits to G_M^{u-d} ($m_\pi = 297$ MeV)

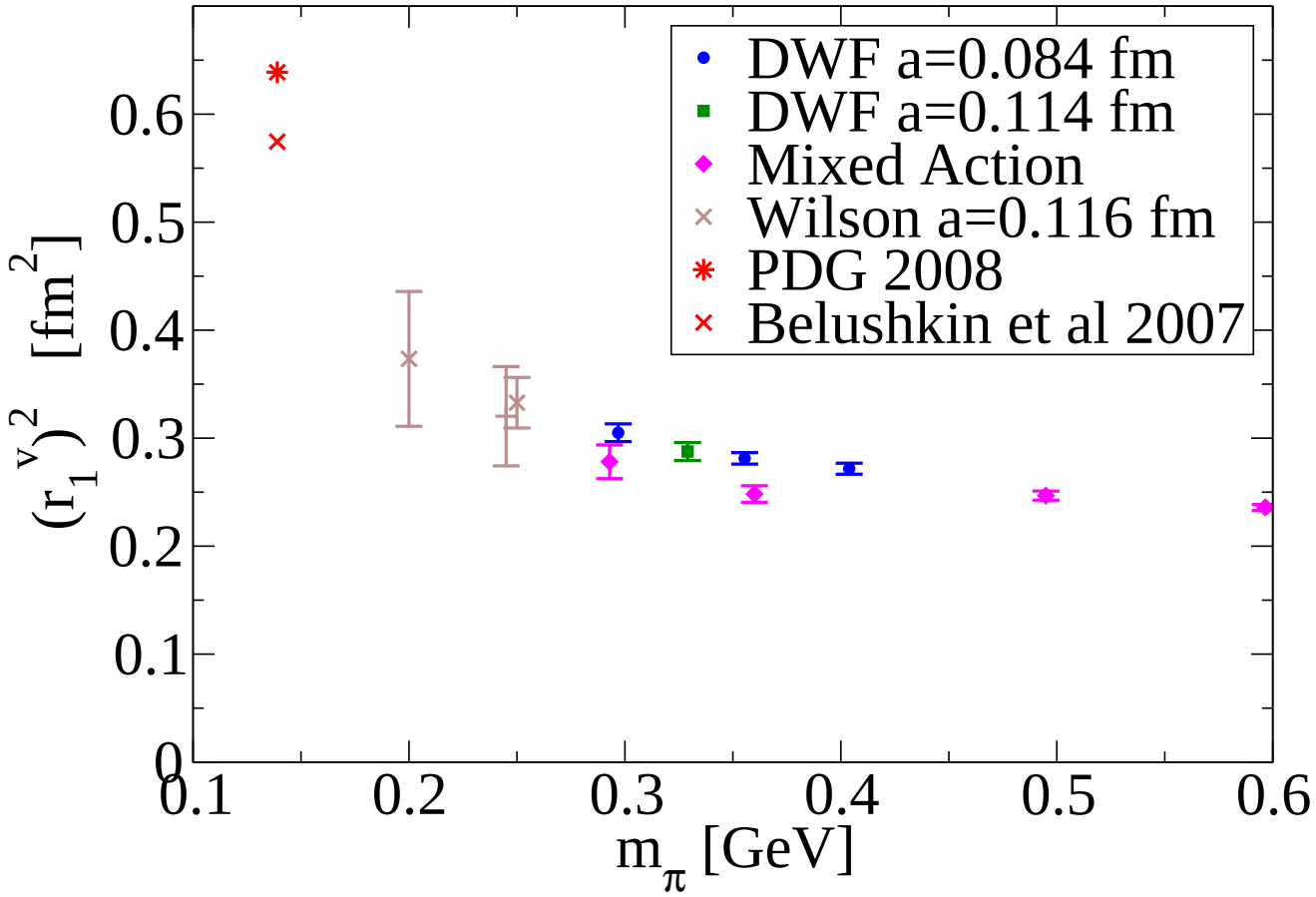


- Dipole fits
  $G_M(Q^2) = \frac{1+\kappa_v}{(1+Q^2/M_D^2)^2}$
- Cut-off $Q^2 \leq 0.5$ GeV^2
- Results are consistent for higher cutoffs

$M_D^2 = 0.97 \ldots 1.07$ GeV^2

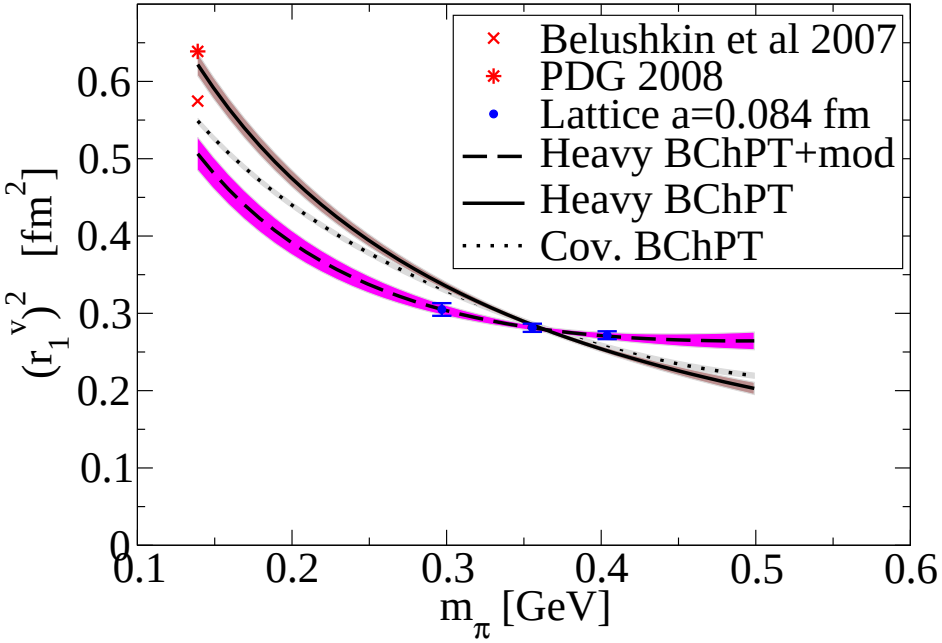
$(M_D^2)_{\text{exp}} = 0.71$ GeV^2

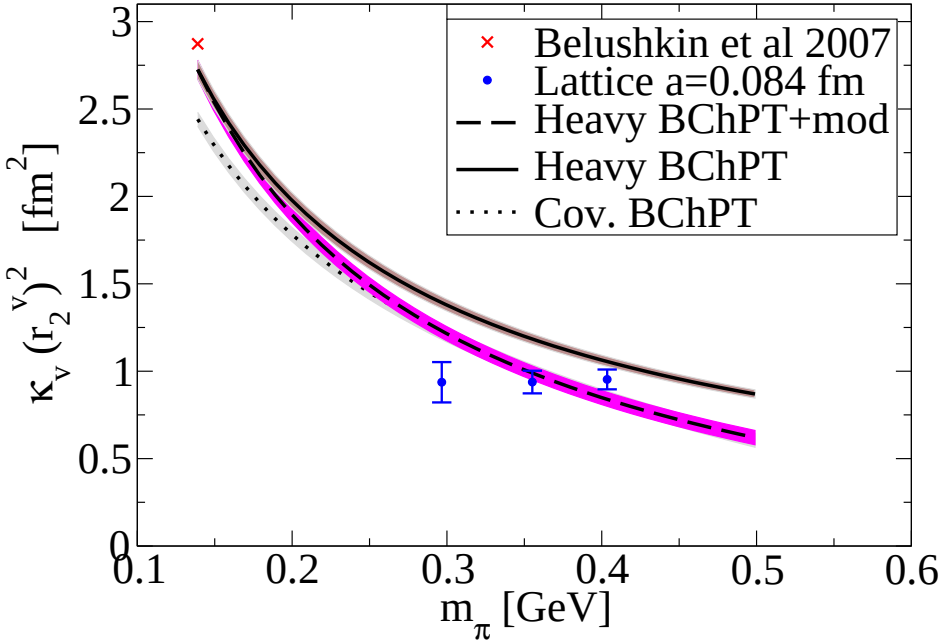
Dirac Radius $\langle r_1^2 \rangle^{u-d}$



$$F_1(Q^2) = 1 - \frac{1}{6}\langle r_1^2 \rangle \cdot Q^2 + O(Q^4)$$

Dirac and Pauli Radii vs. ChPT





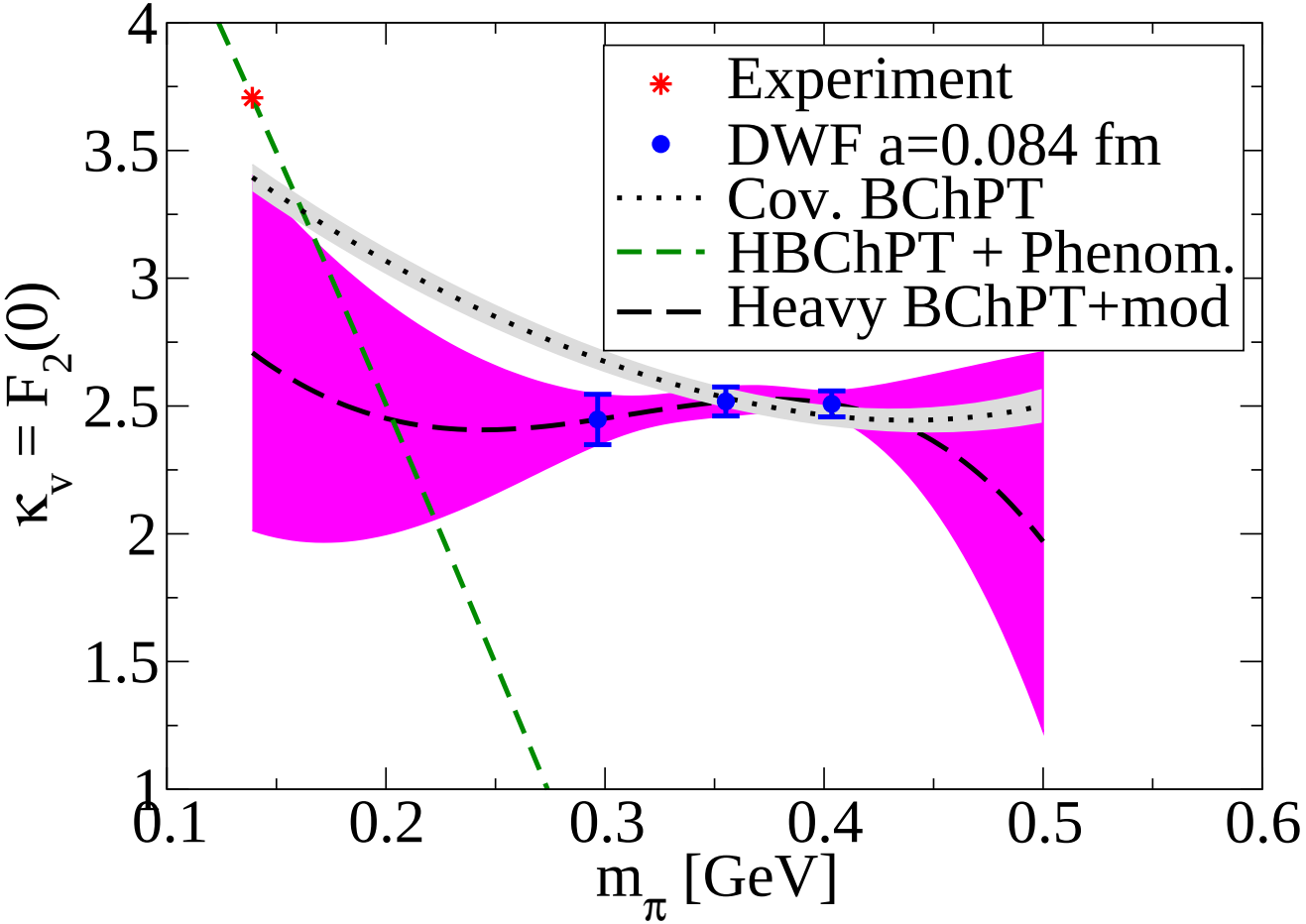
Div. "pion cloud" contribution:

$$\delta\left[(r_1^v)^2\right] \sim \log m_\pi$$
$$\delta\left[\kappa_v (r_2^v)^2\right] \sim \frac{1}{m_\pi}$$

HBChPT [V. Bernard, H. Fearing, T. Hemmert, U.-G. Meissner (1998)]

CBChPT [T. Gail, PhD Thesis (2007)]

Anomalous Magnetic Moment κ_v



HBChPT [V. Bernard, H. Fearing, T. Hemmert, U.-G. Meissner (1998); T. Hemmert, W. Weise (2002)]

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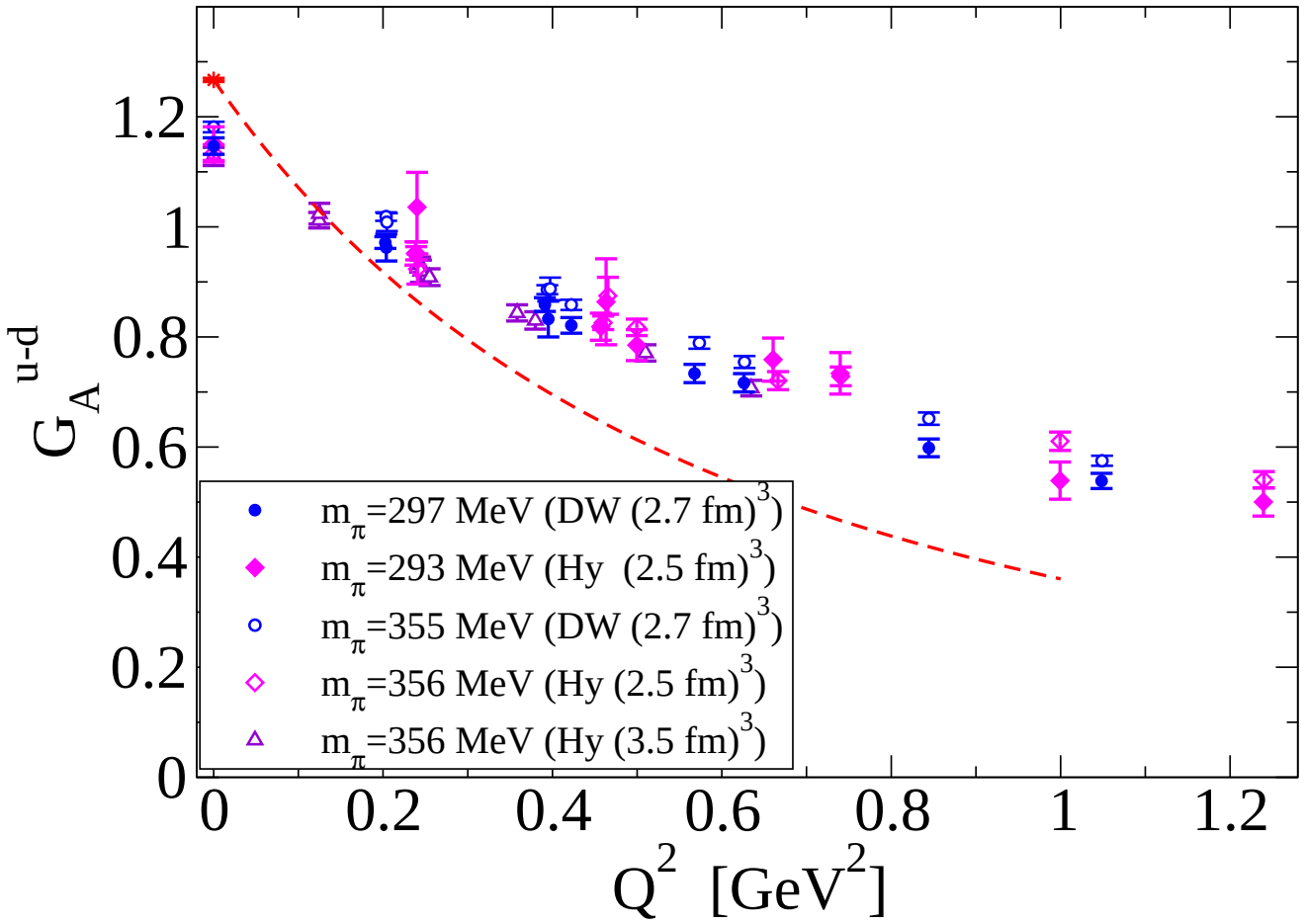
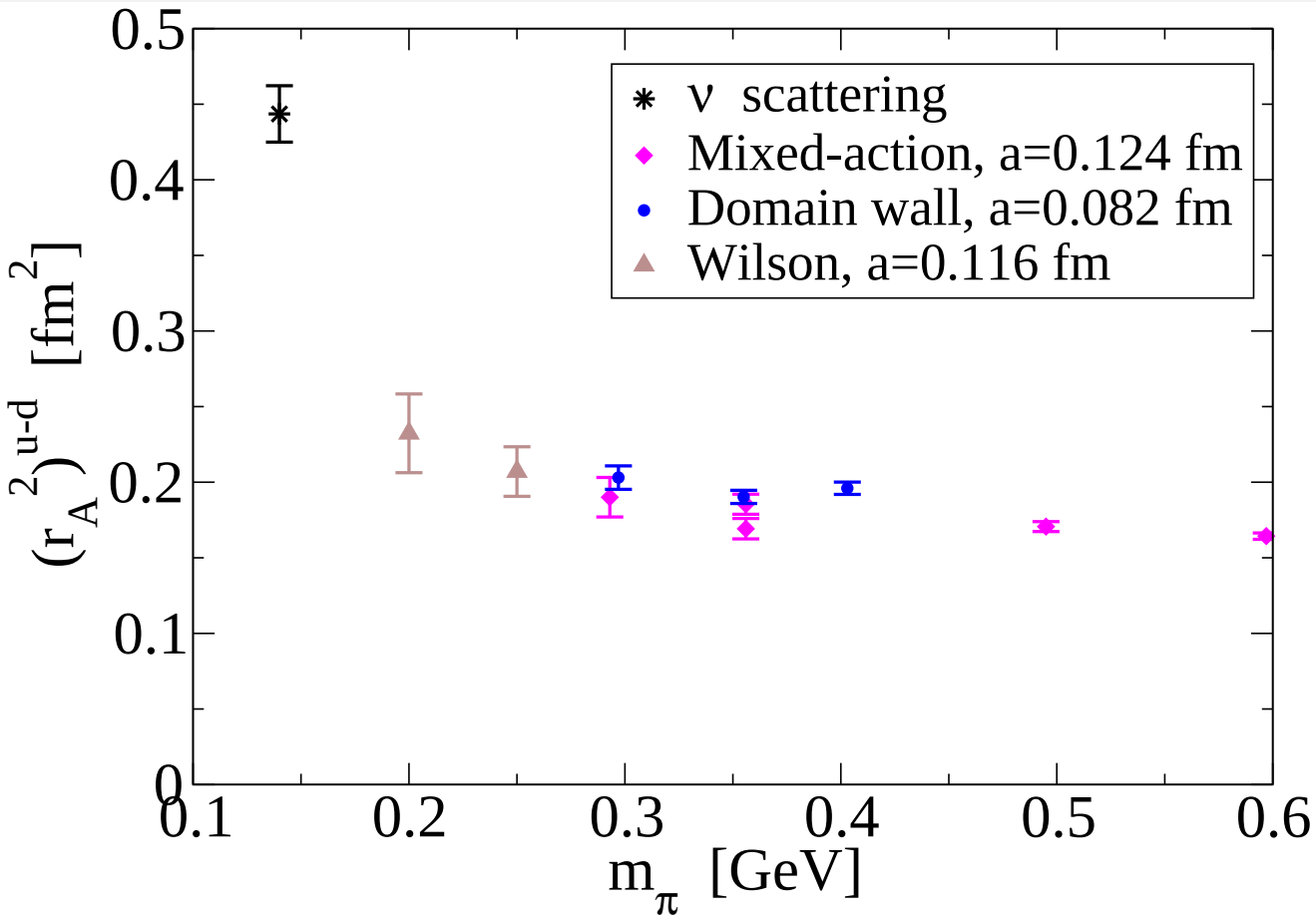
Q^2-dependence of Axial Form Factor G_A^{u-d}



Phenomenology: $G_A(Q^2) = \frac{g_A}{(1+Q^2/M_A^2)^2}$

$M_A = (1.026 \pm 0.021)$ GeV (ν scattering, [PDG 2000])

Axial radius

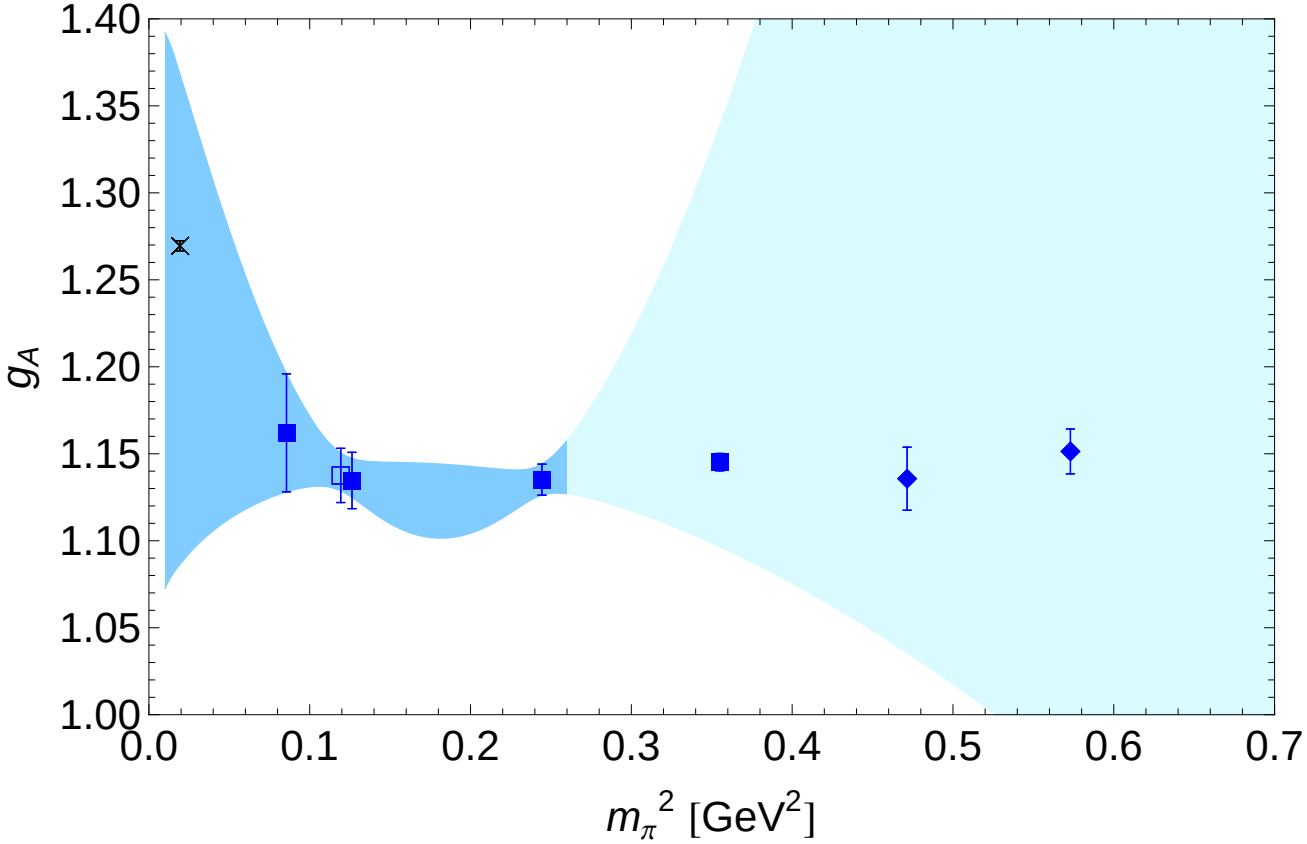


$$\langle r_A^2 \rangle = -\frac{1}{6G_A}\frac{dG_A}{dQ^2}\bigg|_{Q^2=0} = \frac{1}{12}\frac{1}{M_A^2}$$

NLO Heavy Baryon ChPT: $\langle r_A^2 \rangle$ does not depend on m_π

[V. Bernard, H. Fearing, T. Hemmert, U.-G. Meissner (1998)]

Axial Charge $g_A = G_A^{u-d}(0)$

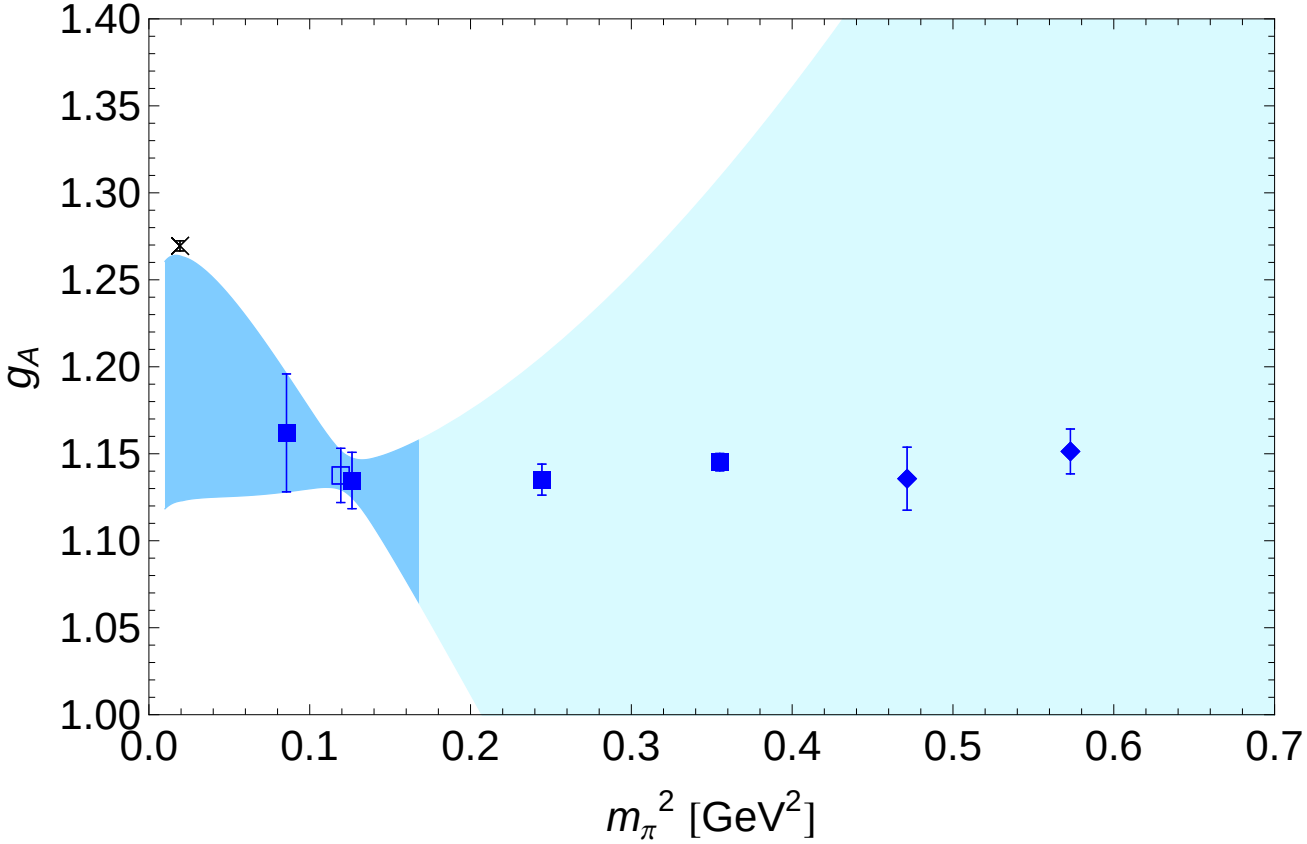


SSE fit [T. Hemmert, M. Procura, W. Weise (2003)]

$m_\pi \lesssim 500$ MeV with 3 parameters: g_A^0, g_1, *counterterm*

[fit by P. Hägler]

Axial Charge $g_A = G_A^{u-d}(0)$

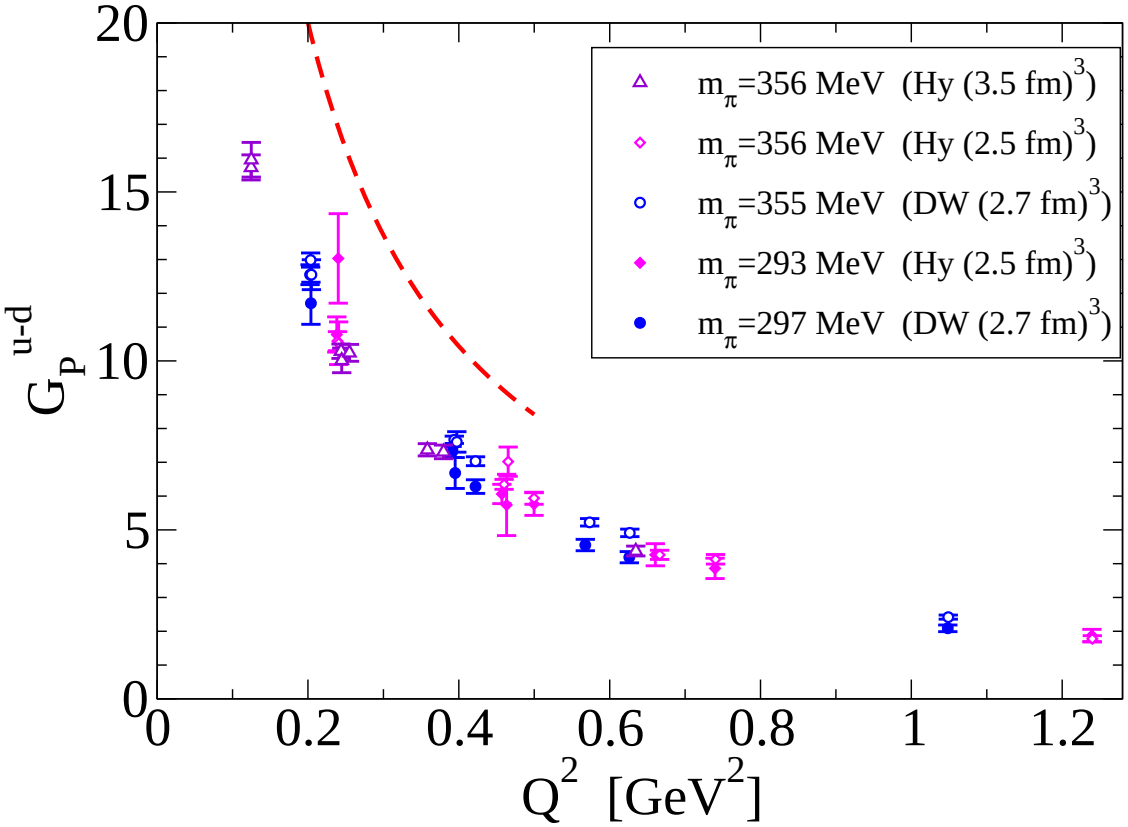


SSE fit [T. Hemmert, M. Procura, W. Weise (2003)]

$m_\pi \lesssim 360$ MeV with 2 parameters: g_A^0, *counterterm*

[fit by P. Hägler]

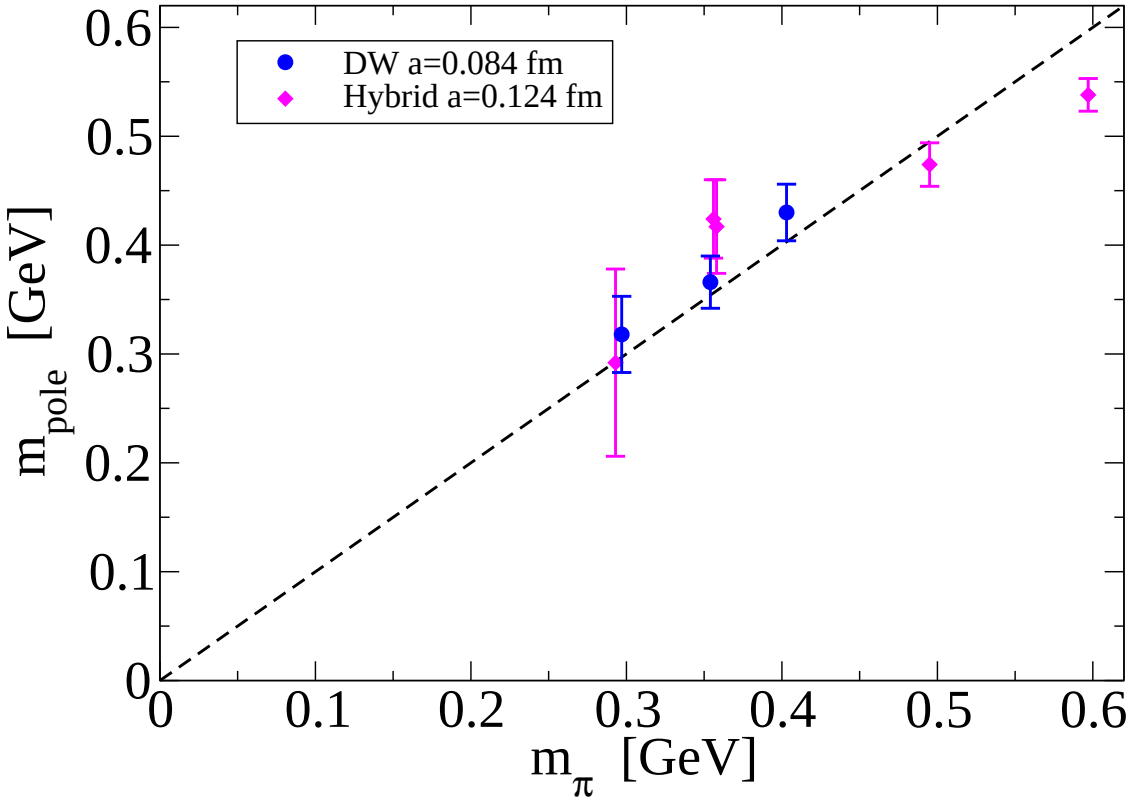
Q^2-dependence of Pseudoscalar G_P^{u-d}



From NLO ChPT $\boxed{G_P(Q^2) = \frac{2m_N f_\pi g_{\pi NN}}{m_\pi^2+Q^2} - \frac{2}{3} g_A m_N^2 \langle r_A^2 \rangle}$

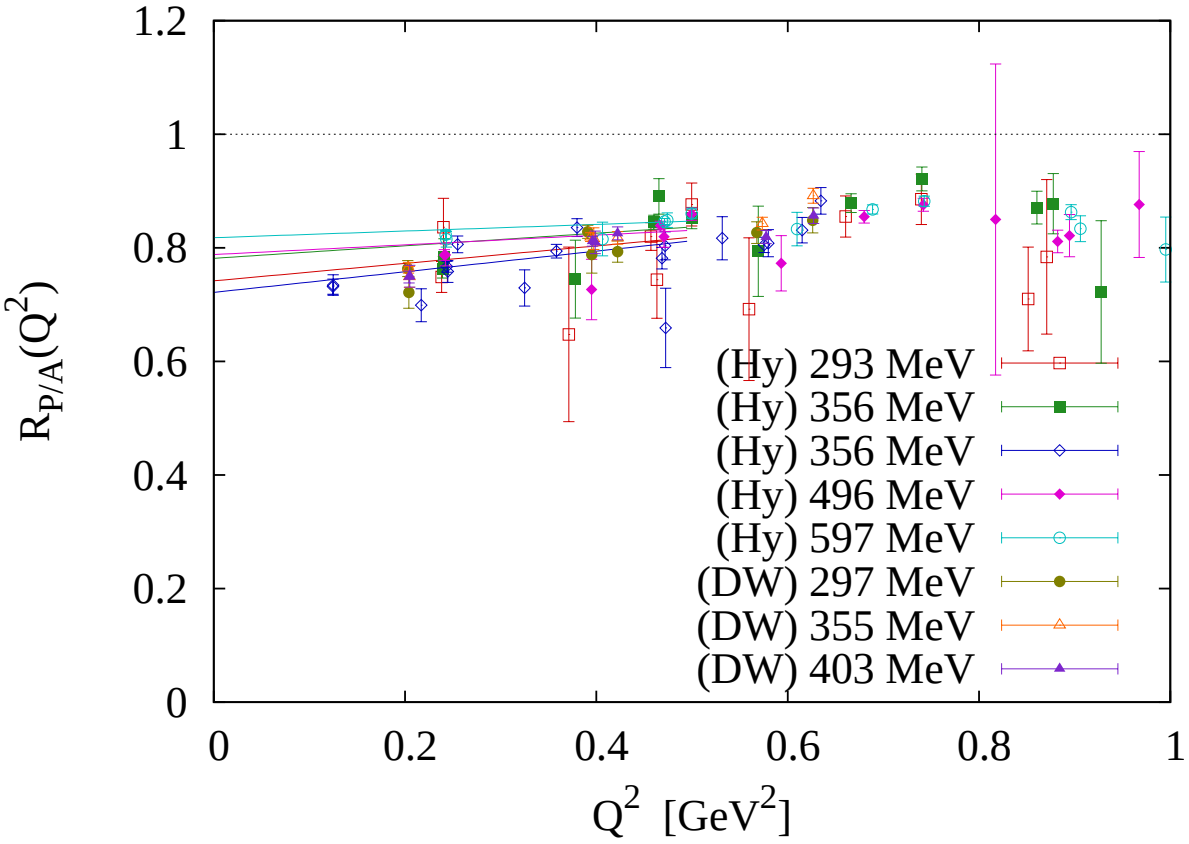
[V. Bernard, H. Fearing, T. Hemmert, U.-G. Meissner (1998)]

Check Pion-pole Dominance



Fit $\boxed{G_P(Q^2) \Longleftrightarrow \frac{a}{m_{\text{pole}}^2+Q^2} + c}$ for $0.1 \leq Q^2 \leq 0.5$ GeV^2

Check Pion-pole Dominance (2)



$$R_{P/A}(Q^2) = \frac{m_\pi^2+Q^2}{4m_N^2} \cdot \frac{G_P(Q^2)}{G_A(Q^2)} \xrightarrow[Q^2\to 0]{} \frac{g_{\pi NN}}{g_{\pi NN}^{\mathsf{GT}}}$$

From HBChPT expect $\frac{g_{\pi NN}}{g_{\pi NN}^{\mathsf{GT}}} = 1 - A \cdot m_\pi^2$

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Outlook

- Accurate lattice data with *chiral quarks* exists for $m_\pi^2 \gtrsim 300$ MeV
- Form factors reproduce qualitatively the expected empirical/phenomenological behavior
  - dipole form for G_E, G_M, G_A
  - pion pole form for G_P
- "Nucleon size" observables $(r_1^2)^v$, $(r_2^2)^v$, $(r_A^2)^v$ undershoot experimental values and weakly depend on m_π
- Calculations at smaller m_π and Q^2 are necessary
  - $m_\pi = 180$ and 250 MeV are underway
  - small Q^2: larger volumes OR twisted BC

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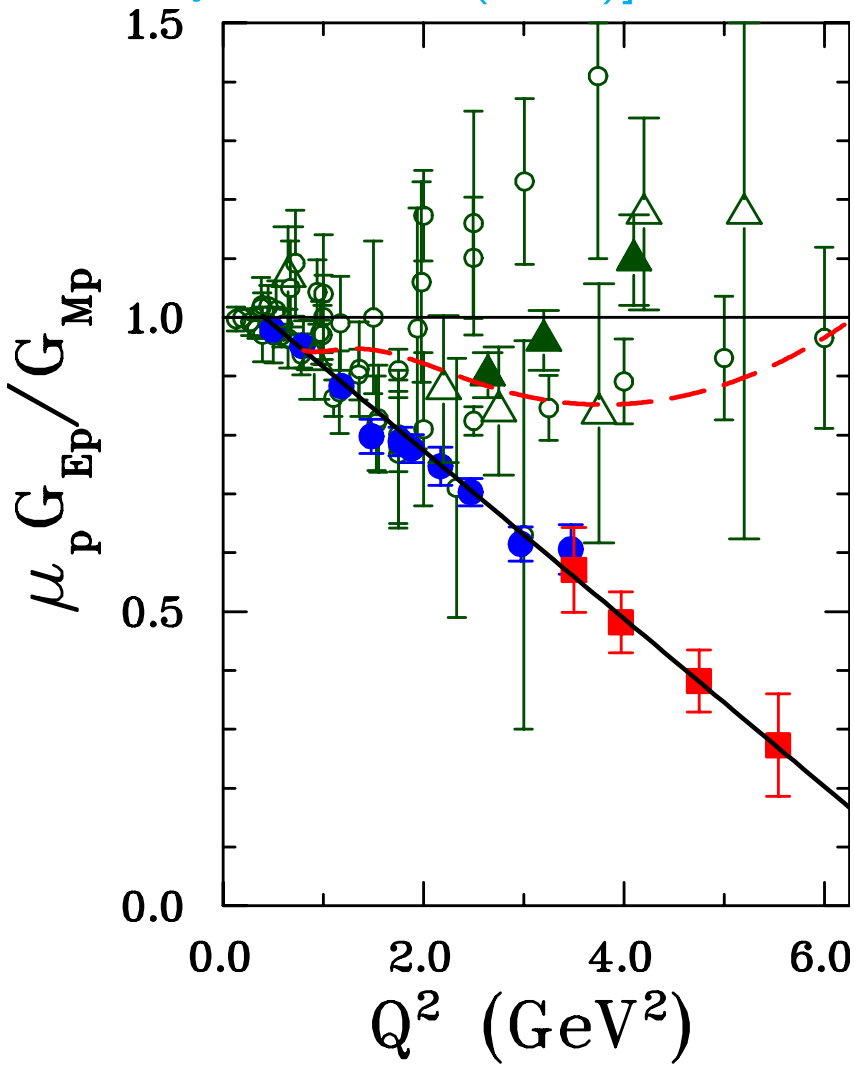
Proton Form Factors

[M. Vanderhaeghen, Nucl.Phys.A805:210(2008)]



Different results from

- Rosenbluth separation
- Polarization transfer

due to 2γ exchange

[Blunden, Melnitchouk and Tjon, 2003]

[Chen et al, 2004]

Proton Form Factors

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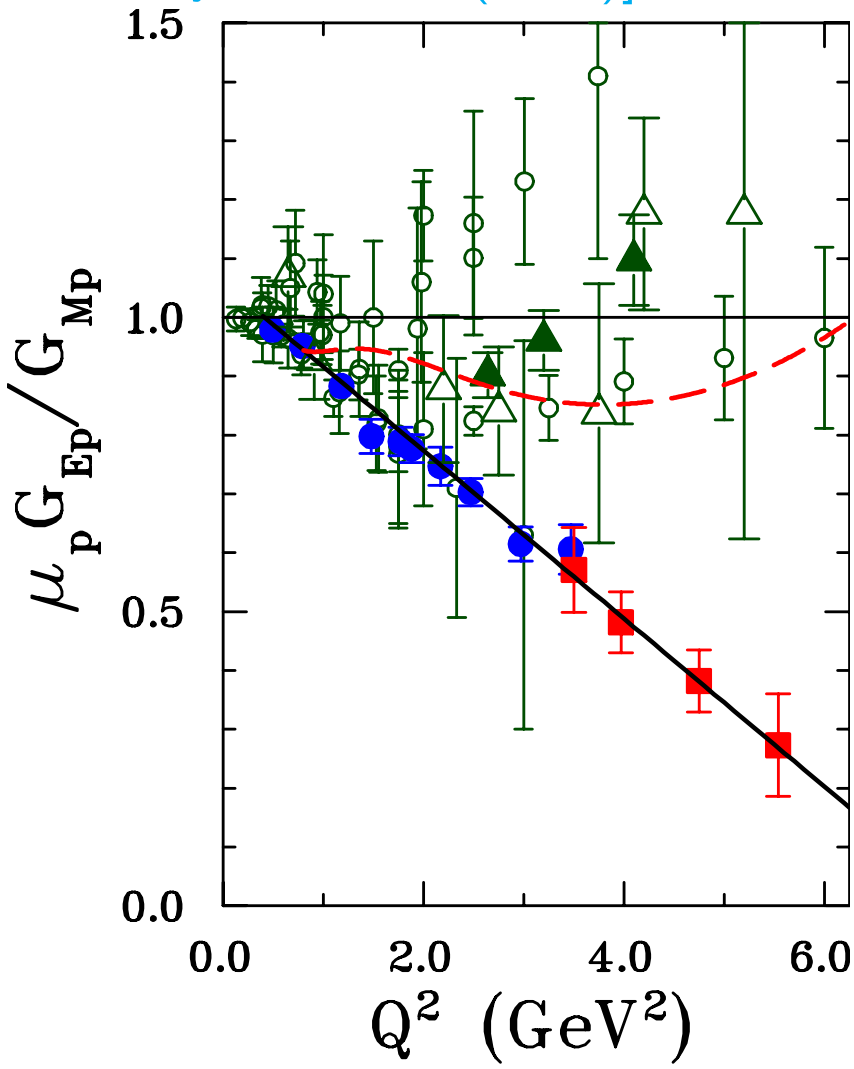

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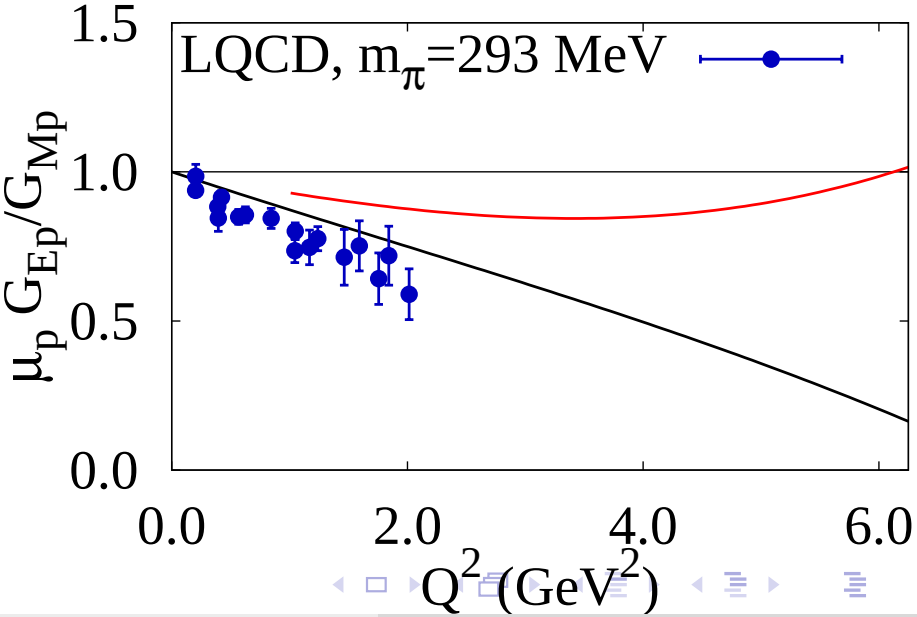
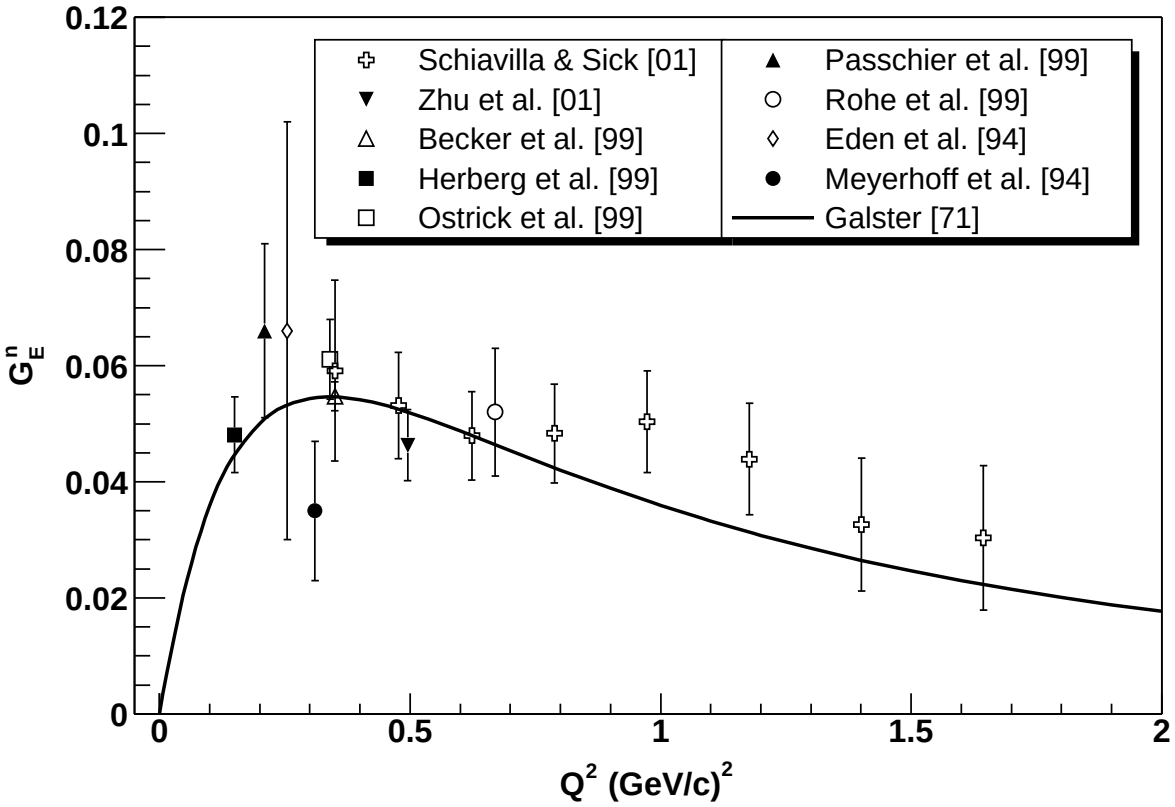
Lattice (systematics unclear!):



Neutron Electric Form Factor



[H. Gao, Int.J.Mod.Phys.E12:1 (2003)]

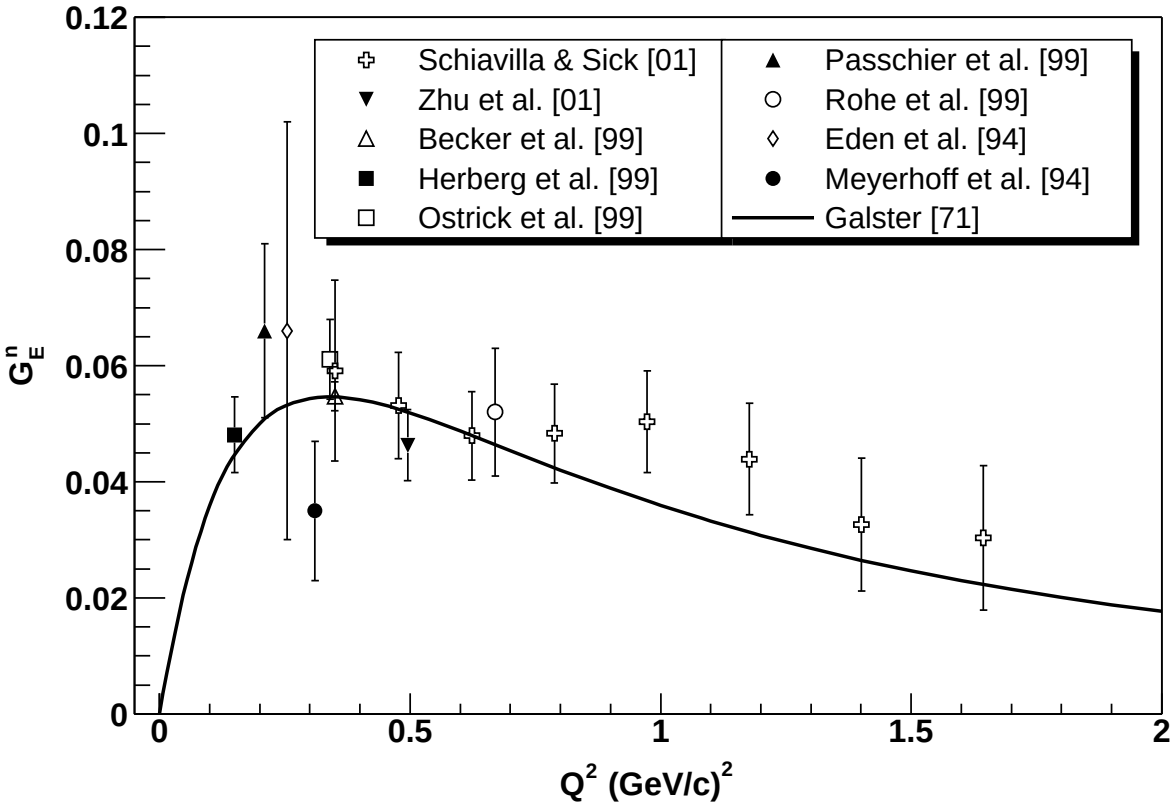
- deuterium targets
- thermal neutron scattering:
  $\langle r^2_{E,n} \rangle = -0.113(3)(4)$ fm^2
  [PRL 74, 2427 (1995)]

$\langle r^2_{En} \rangle < 0$:

$(+)$ core

$(-)$ surface

Neutron Electric Form Factor



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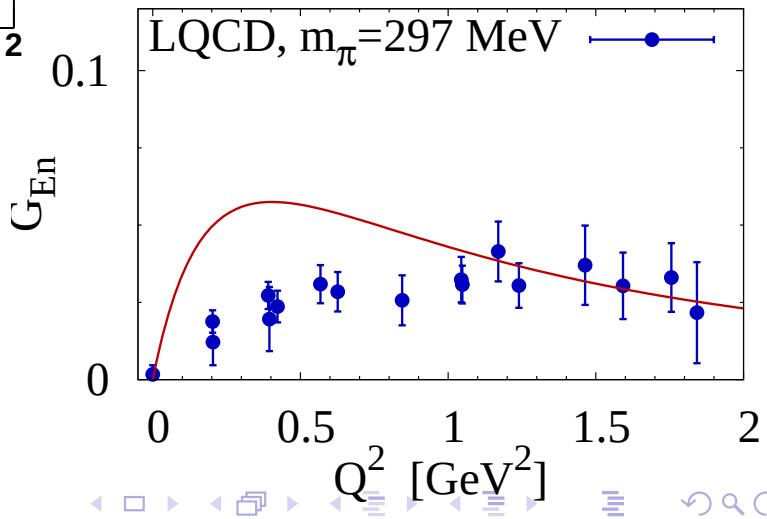

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